

# MATHEMATICS

## UNIT I: ALGEBRA

### SETS, RELATIONS AND FUNCTIONS

Sets and their Representations: Finite and Infinite sets; Empty set; Equal sets; Subsets; Power set; Universal set; Venn Diagrams; Complement of a set; Operations on Sets (Union, Intersection and Difference of Set); Applications of sets: Ordered Pairs, Cartesian Product of Two sets; Relations: Domain, Co-domain and Range: Functions: into, on to, one - one in to, one-one on to Functions; Constant Function; Identity Function; composition of Functions; Invertible Functions; Binary Operations.

#### Complex Numbers

Complex Numbers in the form  $a + ib$ ; Real and Imaginary Parts of a complex Number; Complex Conjugate, Argand Diagram, Representation of Complex Number as a point in the plane; Modulus and Argument of a Complex Number; Algebra of Complex Numbers; Triangle Inequality;

$$a + ib$$

$|Z_1 + Z_2| \leq |Z_1| + |Z_2|$ ;  $|Z_1 \cdot Z_2| = |Z_1| |Z_2|$ ; Polar Representation of a Complex Number.

#### Quadratic Equations

Solution of a Quadratic Equation in the Complex Number System by (i) Factorization (ii) Using Formula; Relation between Roots and Coefficients; Nature of Roots; Formation of Quadratic Equations with given Roots; Equations Reducible to Quadratic Forms.

#### Sequences and Series

Sequence and Examples of Finite and Infinite Sequences; Arithmetic Progression (A.P): First Term, Common Difference,  $n^{\text{th}}$  Term and sum of  $n$  terms of an A.P.; Arithmetic Mean (A.M); Insertion of Arithmetic Means between any Two given Numbers; Geometric Progression (G.P): first Term, Common Ratio and  $n^{\text{th}}$  term, Sum to  $n$  Terms, Geometric Mean (G.M); Insertion of Geometric Means between any two given Numbers.

#### Permutations, Combinations, Binomial Theorem and Mathematical Induction

Fundamental Principle of Counting; The Factorial Notation; Permutation as an Arrangement; Meaning of  $P(n, r)$ ; Combination: Meaning of  $C(n, r)$ ; Applications of Permutations and Combinations. Statement of Binomial Theorem; Proof of Binomial Theorem for positive integral Exponent using Principle of Mathematical Induction and also by combinatorial Method; General and Middle Terms in Binomial Expansions; Properties of Binomial Coefficients; Binomial Theorem for any Index (without proof); Application of Binomial Theorem. The Principle of Mathematical Induction, simple Applications.

#### Matrices and Determinants

Concept of a Matrix; Types of Matrices; Equality of Matrices (only real entries may be considered): Operations of Addition, Scalar Multiplication and Multiplication of Matrices; Statement of Important Results on operations of Matrices and their Verifications by Numerical Problem only; Determinant of a Square Matrix; Minors and Cofactors; singular and non-singular Matrices; Applications of Determinants in (i) finding the Area of a Triangle (ii) solving a system of Linear Equations (Cramer's Rule); Transpose, Adjoint and Inverse of a Matrix; Consistency and Inconsistency of a system of Linear Equations; Solving

System of Linear Equations in Two or Three variables using Inverse of a Matrix (only up to  $3 \times 3$  Determinants and Matrices should be considered).

#### Linear Inequations

Solutions of Linear Inequation in one variable and its Graphical Representation; solution of system of Linear Inequations in one variable; Graphical solutions of Linear inequations in two variables; solutions of system of Linear Inequations in two variables.

#### Mathematical Logic and Boolean Algebra

Statements; use of Venn Diagram in Logic; Negation Operation; Basic Logical Connectives and Compound Statements including their Negations.

## UNIT II : TRIGONOMETRY

### Trigonometric functions and Inverse Trigonometric functions

Degree measures and Radian measure of positive and negative angles; relation between degree measure and radian measure, definition of trigonometric functions with the help of a unit circle, periodic functions, concept of periodicity of trigonometric functions, value of trigonometric functions

of  $x$  for  $x = 0, \pi$

$$\pi, 4$$

$$\pi, 3$$

$$\pi, 2$$

$\pi, \pi, 3, 2\pi, 2\pi$ ; trigonometric functions of sum and difference of numbers.

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y; \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y; \left( \begin{array}{l} \sin x \tan y \\ \tan x \tan y \end{array} \right)$$

$$\tan x \tan y$$

$$\tan x \tan y \tan x \tan y$$

$$1$$

$$\pm$$

$$\pm$$

$$\pm = ;$$

$$\sin(2\pi \pm x) = \pm \sin x, \cos(2\pi \pm x) = \cos x; \cos(-x) = \cos x, \sin(-x) = -\sin x; \cos(\pi \pm x) = \mp \sin x$$

$$\sin(\pi \pm x) = \mp \sin x; \cos(\pi \pm x) = -\cos x, \sin(\pi \pm x) = \pm \sin x$$

**Trigonometric functions of multiple and submultiples of numbers.**

$$\sin 2x = 2 \sin x \cos x;$$

$$\sin 3x = 3 \sin x - 4 \sin^3 x; \cos 2x = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1; \cos 3x = 4 \cos^3 x - 3 \cos x$$

$$\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}; \sin x + \sin y = 2 \sin \left( \frac{x+y}{2} \right) \cos \left( \frac{x-y}{2} \right); \cos x + \cos y = 2 \cos \left( \frac{x+y}{2} \right) \cos \left( \frac{x-y}{2} \right)$$

$$\sin x - \sin y = 2 \cos \left( \frac{x+y}{2} \right) \sin \left( \frac{x-y}{2} \right); \cos x - \cos y = -2 \sin \left( \frac{x+y}{2} \right) \sin \left( \frac{x-y}{2} \right)$$

Conditional identities for the angles of a triangle, solution of trigonometric equations of the type  $\sin x = \sin a$ ;  $\cos x = \cos a$ ;  $\tan x = \tan a$  and equations reducible to these forms.

**Inverse Trigonometric functions:**

(i)  $\sin^{-1}(\sin x) = x$  and other similar formula (ii)  $\sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1} x$  and other similar formula.

$$\sin^{-1}(-x) = -\sin^{-1} x, \tan^{-1}(-x) = -\tan^{-1} x; \operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x, \cos^{-1}(-x) = \pi - \cos^{-1}(x);$$

$$\sec^{-1}(-x) = \pi - \sec^{-1}(x), \cot^{-1}(-x) = \pi - \cot^{-1}(x)$$

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}; \operatorname{cosec}^{-1}(x) + \sec^{-1}(x) = \frac{\pi}{2}; \tan^{-1} x - \tan^{-1} y = \tan^{-1} \left( \frac{x-y}{1+xy} \right), xy > -1$$

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left( \frac{x+y}{1-xy} \right), xy < 1; 2 \tan^{-1} x = \sin^{-1} \left( \frac{2x}{1+x^2} \right) = \cos^{-1} \left( \frac{1-x^2}{1+x^2} \right) = \tan^{-1} \left( \frac{2x}{1-x^2} \right), |x| < 1$$

**Simple problems**

Graph of the following trigonometric functions;

$$y = \sin x; y = \cos x; y = \tan x; y = a \sin x; y = a \cos x; y = a \sin bx; y = a \cos bx;$$

### UNIT III: GEOMETRY

#### Cartesian System of Rectangular Co ordinates

Cartesian system of co ordinates in a plane, Distance formula, Centroid and incentre, Area of a triangle, condition for the collinearity of three points in a plane, Slope of line, parallel and perpendicular lines, intercepts of a line on the co ordinate axes, Locus and its equation.

## Lines and Family of lines

Various forms of equations of a line parallel to axes, slope-intercept form, The Slope point form, Intercept form, Normal form, General form, Intersection of lines. Equation of bisectors of angle between two lines, Angles between two lines, condition for concurrency of three lines, Distance of a point from a line, Equations of family of lines through the intersection of two lines.

## Circles and Family of circles

Standard form of the equation of a circle General form of the equation of a circle, its radius and center, Equation of the circle in the parametric form.

## Conic sections

Sections of a cone. Equations of conic sections [Parabola, Ellipse and Hyperbola] in standard form.

## Vectors

Vectors and scalars, Magnitude and Direction of a vector, Types of vectors (Equal vectors, unit vector, Zero vector). Position vector of a point, Localized and free vectors, parallel and collinear vectors, Negative of a vector, components of a vector, Addition of vectors, multiplication of a vector by a scalar, position vector of point dividing a line segment in a given ratio, Application of vectors in geometry. Scalar product of two vectors, projection of a vector on a line, vector product of two vectors.

## Three Dimensional Geometry

Coordinate axes and coordinate planes in three dimensional space, coordinate of a point in space, distance between two points, section formula, direction cosines, and direction ratios of a line joining two points, projection of the join of two points on a given line, Angle between two lines whose direction ratios are given, Cartesian and vector equation of a line through (i) a point and parallel to a given vector (ii) through two points, Collinearity of three points, coplanar and skew lines, Shortest distance between two lines, Condition for the intersection of two lines, Cartesian and vector equation of a plane (i) When the normal vector and the distance of the plane from the origin is given (ii) passing through a point and perpendicular to a given vector (iii) Passing through a point and parallel to two given lines through the intersection of two other planes (iv) containing two lines (v) passing through three points, Angle between (i) two lines (ii) two planes (iii) a line and a plane, Condition of coplanarity of two lines in vector and Cartesian form, length of perpendicular of a point from a plane by both vector and Cartesian methods.

## UNIT IV: STATISTICS

### Statistics and probability

Mean deviation for ungrouped data, variance for grouped and ungrouped data, standard deviation. Random experiments and sample space, Events as subset of a sample space, occurrence of an event, sure and impossible events, Exhaustive events, Algebra of events, Meaning of equally likely outcomes, mutually exclusive events. Probability of an event; Theorems on probability; Addition rule, Multiplication rule, Independent experiments and events. Finding  $P(A \text{ or } B)$ ,  $P(A \text{ and } B)$ , random variables, Probability distribution of a random variable.

## UNIT V : CALCULUS

### Functions, Limits and continuity

Concept of a real function; its domain and range; Modulus Function, Greatest integer function: Signum functions; Trigonometric functions and inverse trigonometric functions and their graphs; composite functions, Inverse of a function.

Limit of a function; meaning and related notations; Left and right hand limits; Fundamental theorems

on limits without proof  $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}, a > 0; \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1; \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$  (without proof);

$\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$  Limits at Infinity and infinity limits; continuity of a function at a point, over an open/ closed interval; Sum, Product and quotient of continuous functions; Continuity of special functions- Polynomial, Trigonometric, exponential, Logarithmic and Inverse trigonometric functions.

## Differentiation

Derivative of a function; its geometrical and physical significance; Relationship between continuity and differentiability; Derivatives of polynomial, basic trigonometric, exponential, logarithmic and inverse trigonometric functions from first principles; derivatives of sum, difference, product and quotient of functions; derivatives of polynomial, trigonometric, exponential, logarithmic, inverse trigonometric and implicit functions; Logarithmic differentiation; derivatives of functions expressed in parametric form; chain rule and differentiation by substitution; Derivatives of Second order.

## Application of Derivatives

Rate of change of quantities; Tangents and Normals; increasing and decreasing functions and sign of the derivatives; maxima and minima; Greatest and least values; Rolle's theorem and Mean value theorem; Approximation by differentials.

## Indefinite Integrals

Integration as inverse of differentiation; properties of integrals; Integrals involving algebraic trigonometric, exponential and logarithmic functions; Integration by substitution; Integration by parts  
Integrals of the type:

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{a^2 - x^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c},$$
$$\int \frac{px+q}{ax^2 + bx + c} dx, \int \frac{dx}{\sqrt{ax^2 + bx + c}}, \int \frac{px+q}{\sqrt{ax^2 + bx + c}} dx.$$

Integration of rational functions; Partial fractions and their use in integration; Integrals of the type

$$\int \sqrt{x^2 \pm a^2} dx, \int \sqrt{a^2 - x^2} dx, \int \sqrt{(ax^2 + bx + c)} dx, \int (px + q)\sqrt{(ax^2 + bx + c)} dx,$$
$$\int \frac{dx}{a + b \cos x}, \int \frac{dx}{a - b \sin x}, \int \sin^{-1} x dx, \int \log x dx.$$

## Definite Integrals

Definite integral as limit of a sum; Fundamental theorems of integral calculus without proof); Evaluation of definite integrals by substitution and by using the following properties.

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{a^2 - x^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c},$$
$$\int \frac{px+q}{ax^2 + bx + c} dx, \int \frac{dx}{\sqrt{ax^2 + bx + c}}, \int \frac{px+q}{\sqrt{ax^2 + bx + c}} dx.$$

Integration of rational functions; Partial fractions and their use in integration; Integrals of the type

$$\int \sqrt{x^2 \pm a^2} dx, \int \sqrt{a^2 - x^2} dx, \int \sqrt{(ax^2 + bx + c)} dx, \int (px + q)\sqrt{(ax^2 + bx + c)} dx,$$
$$\int \frac{dx}{a + b \cos x}, \int \frac{dx}{a - b \sin x}, \int \sin^{-1} x dx, \int \log x dx.$$

Application of definite integrals in finding areas bounded by a curve, circle, parabola and ellipse in standard form between two ordinates and x-axis; Area between two curves, line and circle; line and parabola: line and ellipse.

## Differential Equations

Definition; order and degree; general and particular solutions of a differential equation; formation of differential equations whose general solution is given; solution of differential equations by method of Separation of variables; Homogeneous differential equations of first order and their solutions; Solution of linear differential equations of the type  $P(x)y' + Q(x)y = R(x)$

$dx$

$dy + =$  where  $P(x)$ ,  $Q(x)$  are functions of  $x$ .